The Occurrence of a Finite Number of Particle Families in the Planck Aether Model of a Unified Field Theory

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Z. Naturforsch. **46a,** 677–685 (1991); received June 8, 1990

The Planck aether substratum model explains Dirac spinors as excitons made up from vortex resonances of the positive and negative mass substratum. In the framework of this model it is conjectured that the higher particle families arise from excited states due to radial oscillations. The characteristic feature of quantum mechanics involving negative masses leads to a finite number of such excited states. Without the knowledge of the mass distribution in the excitons, a maximum number of four families can be predicted from the known mass spectrum of the leptons.

Finally, it is conjectured that the small neutrino-electron mass ratio results from a compensating gravimagnetic effect, leading to a value for this ratio equal to $(m(\text{GUT})/m(\text{Planck}))^2$.

1. Introduction

In the Planck aether model [1] space is densely filled with an equal number of positive and negative Planck masses, to be described by a nonlinear equation for the field operators $\psi_\pm$, which are representative for the positive and negative Planck masses:

$$i\hbar \frac{\partial \psi_\pm}{\partial t} = \mp \frac{\hbar^2}{2m_p} \nabla^2 \psi_\pm + 2\hbar\, c\, r_p^2 (\psi_\pm^\dagger \psi_+ - \psi_-^\dagger \psi_-)\, \psi_\pm, \tag{1.1}$$

$$[\psi_\pm(\boldsymbol{r})\, \psi_\pm^\dagger(\boldsymbol{r}')] = \delta(\boldsymbol{r}-\boldsymbol{r}'),$$

$$[\psi_\pm(\boldsymbol{r})\, \psi_\pm(\boldsymbol{r}')] = [\psi_\pm^\dagger(\boldsymbol{r})\, \psi_\pm^\dagger(\boldsymbol{r}')] = 0\,. \tag{1.2}$$

In (1.1) $m_p = \sqrt{\hbar c/G} \cong 2.2 \times 10^{-5}$ g and $r_p = \sqrt{\hbar G/c^3} \cong 1.6 \times 10^{-33}$ *cm* (G gravitational constant), are the Planck mass and Planck length. Assuming that the temperature of the Planck aether is very low (suggested by the low temperature of the cosmic microwave background radiation), it can be viewed as a two-component superfluid substratum. With an equal number of positive and negative Planck masses, pairs of quantized vortex rings can form out of the Planck aether without the expenditure of energy. With the smallest fluid particle given by the Planck length r_p, and since $m_p r_p c = \hbar$, it follows that the velocity in a quantized vortex reaches the velocity of light c at $r = r_p$, which is the radius of the vortex core.

To determine the ring radius is more difficult, but it can be obtained from drawing an analogy to classical fluid dynamics. There it has been found that for a stable configuration the distance of separation between vortex filaments is several hundred times larger than the core radius [2]. This result can be simply interpreted by the requirement that the average Reynolds number in a vortex lattice must be equal to the universal minimum drag Reynolds number [1]. The transition to a quantum fluid is done by replacing the ordinary viscosity with a quantum viscosity as it follows from Schrödinger's wave equation. For curved vortex rings the averaged Reynolds number is larger, because compared with a linear vortex, the velocity in a ring vortex is larger. Applied to a lattice of vortex rings, the ring radius and separation of rings in a vortex lattice is for this reason about 5000 times larger than the vortex core radius. It can be shown that small amplitude waves propagating through this vortex lattice have the same properties as those derived from Maxwell's equations. The vortex rings are coupled to each other by compression waves having their source in the zero point fluctuations of the Planck masses bound in the vortex filaments, with the coupling constants turning out to be equal to Newton's gravitational constant. The shortest wave length for the transverse Maxwell-type waves propagating through the vortex lattice must be about equal to the ring radius r_0. Quantum mechanically, this ring radius corresponds to an energy $\hbar c/r_0 \sim 10^{15}$–10^{16} GeV, which turns out to be about equal the grand unified energy at which the coupling constants of the strong, weak and electromagnetic interaction, extrapolated to very high energies, become equal.

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Quantum mechanically, all macroscopic forces arise from the transverse waves propagating through the vortex lattice, resulting in electromagnetic forces or forces acting like them. Assuming, then, that all objects, including all fermions, are with regard to these forces in a state of static equilibrium, special relativity follows as a derived dynamic symmetry [3]. Lorentz invariance therefore acts as a dynamic selection principle for all those objects with a lifetime large compared to $r_0/c \sim 10^{-40}$ sec. For all known elementary particles this selection principle is extremely well-satisfied.

The quantized vortex rings in the Planck aether resemble the structures of relativistic string-theories, but we note several important differences: Unlike relativistic strings which have a zero diameter and which in a unique way can only be constructed in a higher dimensional space, the filaments of the ring vortices have a finite diameter and are unique in three-dimensional space. Furthermore, whereas relativistic strings have just one parameter, which is their ring radius, the ring vortices can be described by two parameters, the radius r_p of the vortex core and the ring radius r_0. Since the large dimensionless number r_0/r_p is in reality a quantity derived from the nonlinear property of the superfluid Planck aether expressed through a universal quantum Reynolds number, which in principle can be derived without making any additional assumptions, the vortex model is therefore reduced to the Planck length alone. Another important property distinguishing the vortex rings from relativistic strings is that the motion of vortices is always *absolute*, taking place against a distinguished reference system in which the superfluid Planck aether is at rest. It is for this reason that the Planck aether model implies by necessity a preferred reference system, with the Galilei group as the fundamental kinematic symmetry.

In the Planck aether model Dirac spinors are explained as excitons made up from the positive and negative resonance energy of the ring vortices, associated with an oscillating elliptic deformation of the rings, having a resonance energy given by

$$m_v^{\pm} c^2 \cong \pm m_p c^2 (r_p/r_0)^2 \cong \pm 10^{11}\,\text{GeV}. \tag{1.3}$$

In the proposed model, Dirac spinors are a superposition of a very large ($\cong +10^{11}$ GeV) positive mass, with a very large ($\cong -10^{11}$ GeV) negative mass, whereby the resulting spinor mass is the positive gravitational interaction energy between these two large but opposite masses. The gravitational interaction itself results in our model from the mechanical compression waves having their source in the zero point fluctuations of the Planck masses bound in the vortex filaments. Approximating the exciton made up from the positive and negative energy vortex resonances by two pointlike masses of opposite sign, but with the positive mass slightly larger than the negative mass to account for the positive gravitational interaction energy, the very large mass ratio of the Planck mass to the spinor mass ($\approx 10^{22}$) can be derived [1]. The replacement by two pointlike masses, opposite in sign but of slightly different magnitude, is most certainly a too simple approximation to compute higher excited states which, if they exist, are likely to be associated with higher particle families. Because the particles of the higher families have the same angular momentum as their lower energy counterpart, they are probably higher excited states resulting from radial pulsations. Flavor changing transitions between different particle families would then be nonrelativistically forbidden $O^+ \to O^+$ transitions, a property which would explain their rarity.

The determination of the higher excited states requires the knowledge of the form factors for the internal mass distribution of the vortex resonances and their interaction energy. To obtain these form factors is very difficult, but one can also go the opposite way, by trying to obtain the form factors from the known mass spectrum of the first three families, with the goal to obtain in this indirect way some information about possible higher families and their masses. We have chosen here this second indirect approach.

2. Pole-Dipole Model

The vortex resonance energy (1.3) has the masses

$$m_v^{\pm} \cong \pm m_p (r_p/r_0)^2. \tag{2.1}$$

An exciton formed from a positive with a negative mass resonance has the gravitational interaction energy

$$m_0 = \frac{G\,|m_v^{\pm}|^2}{c^2 r}, \tag{2.2}$$

where r is the distance of separation of the two masses with opposite sign. The mass m_0 is positive because the gravitational interaction for masses of opposite sign is positive. In the Planck aether model, where the gravitational interaction results form the virtual com-

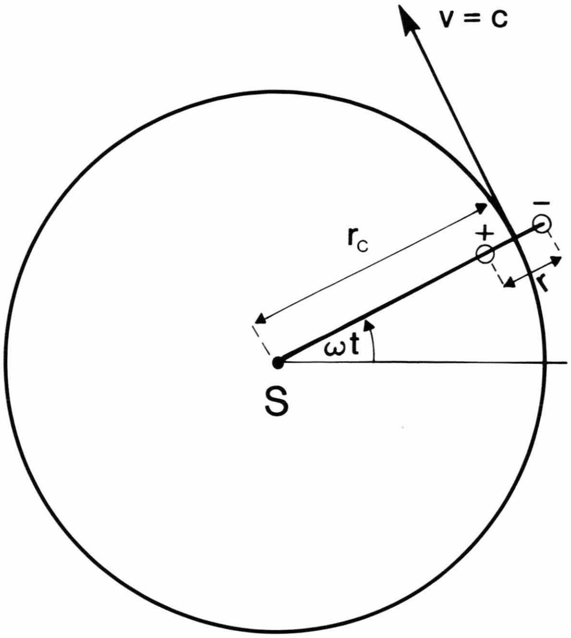

Fig. 1. Circular motion of a pole-dipole particle simulating Schrödinger's Zitterbewegung.

pression waves having their source in the Planck masses bound in the vortex filaments, the interaction energy is the result of an imbalance in the kinetic energy in the flow of the positive and negative mass components of the Planck aether.

Putting $m_v^+ + m_0 = m^+$ and $m_v^- = m^-$, and replacing the configuration consisting of three masses m_v^+, m_v^- and m_0 by two pointlike masses m^+ and m^-, one has what is known as the pole-dipole particle model previously studied by Hönl and Papapetrou [4] (see Figure 1). Momentum conservation implies

$$m^+ \gamma_+ r_c = |m^-| \gamma_-(r_c + r), \tag{2.3}$$

where $\gamma_+ = (1 - v_+^2/c^2)^{-1/2}$, $\gamma_- = (1 - v_-^2/c^2)^{-1/2}$, $v_+ = r_c \omega$, $v_- = (r_c + r)\omega$, with ω the circular orbital frequency around the center of mass. If $m_0 \ll m^+ \cong |m^-|$ one has $r \ll r_c$. For $m^+ > |m^-|$ the distance of the negative mass from the center of mass is larger for m^- than for m^+. For $r \ll r_c$ one can expand (putting $\gamma_+ \equiv \gamma$):

$$\gamma_- = \gamma \left(1 + \frac{r_c r \omega^2 \gamma^2}{c^2} + \ldots\right). \tag{2.4}$$

The mass dipole of the pole-dipole particle is

$$p = m^+ r \cong |m^-| r = \frac{m^+ \gamma - |m^-| \gamma_-}{\gamma_-} r_c, \tag{2.5}$$

which with the help of (2.4) is

$$p \cong m_0 r_c / \gamma^2. \tag{2.6}$$

For the energy of the pole-dipole particle one has

$$E/c^2 = m = m^+ \gamma - |m^-| \gamma_- \cong p\gamma/r_c \tag{2.7}$$

and for its angular momentum (putting $r_c \omega \cong c$)

$$J = [m^+ \gamma r_c^2 - |m^-| \gamma_-(r_c + r)^2]\, \omega \cong -p\gamma c \cong -m c r_c. \tag{2.8}$$

Setting $J = -(1/2)\hbar$,. as required by Dirac's equation, one obtains

$$r_c = \hbar/2mc. \tag{2.9}$$

The minus sign, which means that the angular momentum vector is in opposite direction to the vector $\boldsymbol{\omega}$ of the rotational motion, results from the larger distance of the negative mass form the center of mass. In the pole-dipole model the particle reaches the velocity of light moving in a circle with the radius given by (2.9). It is this orbital motion which produces the spin angular momentum and which turns out to be the same as the "Zitterbewegung" derived by Schrödinger [5] from the Dirac equation, where it results from the negative admixture of energy (and hence mass) states of the Dirac equation.

From (2.6) and (2.7) it follows that

$$m = m_0/\gamma, \tag{2.10}$$

and hence from (2.2):

$$m = \frac{G\,|m_v^\pm|^2}{c^2 \gamma r}. \tag{2.11}$$

From (2.7) and (2.9) with $p \cong |m_v^\pm| r$, one has

$$2\gamma\,|m_v^\pm|\, r c = \hbar. \tag{2.12}$$

Eliminating r from (2.11) and (2.12) one finds

$$m = 2G\,|m_v^\pm|^3/\hbar c = 2\,|m_v^\pm|^3/m_p^2 \tag{2.13}$$

and finally eliminating $|m_v^\pm|$ with the help of (2.1)

$$m/m_p = 2(r_p/r_0)^6. \tag{2.14}$$

Since $m_p r_p c = \hbar$ and $m_G r_0 c = \hbar$, where m_G is the mass at the GUT scale ($m_G c^2 \cong 10^{16}$ GeV), one can also write for (2.14)

$$m/m_p = 2(m_G/m_p)^6. \tag{2.15}$$

Finally, with $m_p = \sqrt{\hbar c/G}$ one can write down the remarkable equation

$$G = (2/m)^2 \hbar c (m_G/m_p)^{12}. \tag{2.16}$$

If the value of m in (2.16) is set equal to the electron mass, one finds that

$$\frac{m_{\mathrm{p}}}{m_{\mathrm{G}}}=\left(\frac{2}{m}\right)^{1/6}\left(\frac{\hbar c}{G}\right)^{1/12}\cong 6000\,, \tag{2.17}$$

a result which is in very good agreement with the value $r_0/r_{\mathrm{p}}\cong 5000$ obtained from hydrodynamic arguments.

3. Lagrange Formalism

The pole-dipole model, which was used to obtain an estimate for the groundstate, it is too simple to approximate the much more complicated structure function for the mass distribution of two overlapping vortex resonances. Fortunately, a generalized Lagrange formalism ideally suited to treat this problem has been developed by Bopp in his field mechanic [6]. One important advantage of Bopp's formalism is that it leaves open the structure function, which otherwise would have first to be determined from the mass distribution of the vortex resonances. This freedom makes it possible to make a fit of the structure function to reproduce the known masses for the different particle families, and from there to predict by extrapolation the existence and masses of particles belonging to higher families.

In the presence of negative masses the Lagrange function must be of the form $L=L(q_k,\dot{q}_k,\ddot{q}_k)$ because even without external forces present, a mass dipole is self-accelerating. To satisfy the dynamic selection principle, Lorentz invariant Lagrange functions must be chosen.

The Euler Lagrange equations of the variational principle

$$\delta\int L(q_k,\dot{q}_k,\ddot{q}_k)\,\mathrm{d}t=0 \tag{3.1}$$

lead to a set of two canonical equations, one for the macrovariables describing the system as a whole, and one for the microvariables describing the Zitterbewegung-type degrees of freedom.

For the relativistic four-vector of the velocity

$$u_a=\mathrm{d}x_a/\mathrm{d}s\equiv\dot{x}_a,\quad \mathrm{d}s=\sqrt{1-\beta^2}\,\mathrm{d}t\,, \tag{3.2}$$

where $\beta=v/c$, $x_a=(x_1,x_2,x_3,ict)$, one has

$$F=u_a^2=-c^2\,. \tag{3.3}$$

With units where $c=1$, one can take instead of (3.1) the variational principle

$$\delta\int\Lambda(x_a,u_a,\dot{u}_a)\,\mathrm{d}s=0\,. \tag{3.4}$$

With (3.3) as a subsidiary condition, the Euler-Lagrange equations, with λ a Lagrange multiplier, are

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\partial(\Lambda+\lambda F)}{\partial u_a}-\frac{\mathrm{d}}{\mathrm{d}s}\frac{\partial\Lambda}{\partial\dot{u}_a}\right)-\frac{\partial\Lambda}{\partial x_a}=0\,. \tag{3.5}$$

For a field-free configuration without external forces, the Lagrange function depends only on $\dot{u}_a$. With the arbitrary function $f(Q)$ one can write for Λ:

$$\Lambda=-f(Q),\quad Q=\dot{u}_a^2 \tag{3.6}$$

with the equation of motion (3.5) taking the form

$$\frac{\mathrm{d}}{\mathrm{d}s}\left\{[f(Q)-4Qf'(Q)]\,u_a+2\frac{\mathrm{d}}{\mathrm{d}s}[f'(Q)\,\dot{u}_a]\right\}=0 \tag{3.7}$$

and the dipole moment equal to

$$p_a=-2f'(Q)\frac{\mathrm{d}u_a}{\mathrm{d}s}\,. \tag{3.8}$$

For the transition to wave mechanics one needs the canonical representation of the equation of motion. From $\int\Lambda\,\mathrm{d}s=\int L\,\mathrm{d}t$ one obtains by separating the space and the time part:

$$L=\Lambda\sqrt{1-v^2}=-f(Q)\sqrt{1-v^2}\,,$$
$$Q=\frac{1}{\sqrt{1-v^2}^{\,4}}\left[\dot{v}^2+\left(\frac{\boldsymbol{v}\cdot\dot{\boldsymbol{v}}}{\sqrt{1-v^2}}\right)^2\right], \tag{3.9}$$

where $L=L(\boldsymbol{r},\dot{\boldsymbol{r}},\ddot{\boldsymbol{r}})$.

With

$$\boldsymbol{P}=\frac{\partial L}{\partial\boldsymbol{v}}-\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial\dot{\boldsymbol{v}}}\,,\quad \boldsymbol{\vartheta}=\frac{\partial L}{\partial\dot{\boldsymbol{v}}} \tag{3.10}$$

one obtains the canonical equations

$$\dot{\boldsymbol{P}}=-\frac{\partial H}{\partial\boldsymbol{r}}\,,\quad \dot{\boldsymbol{r}}=\frac{\partial H}{\partial\boldsymbol{P}}\,,\quad \dot{\boldsymbol{\vartheta}}=-\frac{\partial H}{\partial\boldsymbol{v}}\,,\quad \dot{\boldsymbol{v}}=\frac{\partial H}{\partial\boldsymbol{\vartheta}}\,. \tag{3.11}$$

$\boldsymbol{P}$ and $\boldsymbol{r}$ are here the macrovariables, and $\boldsymbol{\vartheta}$ and $\boldsymbol{v}$ the microvariables. With these variables, the angular momentum conservation law takes the form

$$\boldsymbol{r}\times\boldsymbol{P}+\boldsymbol{v}\times\boldsymbol{\vartheta}=\mathrm{const}\,. \tag{3.12}$$

The first term represents the external angular momentum of the macromotion and the second term the internal spin-type angular momentum of the micromotion. With $\boldsymbol{P}$ and $\boldsymbol{\vartheta}$ given by (3.10) one obtains the

Hamilton function ($P=\{\boldsymbol{\vartheta}, i\vartheta_4\}$)

$$H=\boldsymbol{v}\cdot\boldsymbol{P}+\dot{\boldsymbol{v}}\cdot\boldsymbol{\vartheta}-L=H(\boldsymbol{r},\boldsymbol{P};\boldsymbol{v},\boldsymbol{\vartheta}). \tag{3.13}$$

From (3.10) one finds that

$$\boldsymbol{\vartheta}=-2\frac{f'(Q)}{\sqrt{1-v^2}^3}\left[\dot{\boldsymbol{v}}+\frac{(\boldsymbol{v}\cdot\dot{\boldsymbol{v}})\,\boldsymbol{v}}{1-v^2}\right],$$

$$\dot{\boldsymbol{v}}=-\frac{1}{2}\frac{\sqrt{1-v^2}^3}{f'(Q)}[\boldsymbol{\vartheta}-(\boldsymbol{v}\cdot\boldsymbol{\vartheta})\,\boldsymbol{v}]. \tag{3.14}$$

Eliminating $\dot{\boldsymbol{v}}\cdot\boldsymbol{\vartheta}$ from these equations leads to

$$4Q\,f'(Q)^2=R=(1-v^2)[\boldsymbol{\vartheta}^2-(\boldsymbol{v}\cdot\boldsymbol{\vartheta})^2], \tag{3.15}$$

from which the function $Q=Q(R)$ can be obtained, and by which $\dot{\boldsymbol{v}}$ can be eliminated from H:

$$H=\boldsymbol{v}\cdot\boldsymbol{P}+\sqrt{1-v^2}\,F(R), \tag{3.16}$$

where

$$F(R)=f(Q)-2Q\,f'(Q). \tag{3.17}$$

For the linear dependence

$$f(Q)=k_0+(1/2)\,k_1 Q, \tag{3.18}$$

where k_0 and k_1 are constants, one finds

$$H=k_0\sqrt{1-v^2}-(1/2k_1)\sqrt{1-v^2}^3(\boldsymbol{\vartheta}^2-(\boldsymbol{\vartheta}\cdot\boldsymbol{v})^2), \tag{3.19}$$

which has the same form as the Dirac Hamiltonian.

Putting

$$\boldsymbol{P}\to\frac{\hbar}{i}\frac{\partial}{\partial\boldsymbol{r}},\quad \boldsymbol{v}\to\boldsymbol{a},\quad \sqrt{1-v^2}\to a_4, \tag{3.20}$$

where $\alpha=\{\boldsymbol{\alpha},\alpha_4\}$ are the Dirac matrices, one obtains the Dirac equation

$$\frac{\hbar}{i}\frac{\partial\psi}{\partial t}+H\psi=0, \tag{3.21}$$

where

$$H=a_1P_1+a_2P_2+a_3P_3+a_4 m \tag{3.22}$$

with

$$a_\mu^2=1;\quad a_\mu a_\nu+a_\nu a_\mu=0,\quad \mu\neq\nu, \tag{3.23}$$

and where the mass is given by

$$m=k_0-(1/2k_1)(\boldsymbol{\vartheta}^2-(\boldsymbol{\vartheta}\cdot\boldsymbol{v})^2). \tag{3.24}$$

The introduction of negative masses and invoking Lorentz invariance as a dynamic selection principle, therefore leads to Dirac's equation and sets the spin quantization rule to be $J_z=-(1/2)\,\hbar$, which we had assumed to arrive at (2.9). In the appendix we show that the linear dependence (3.18) leads to the pole-dipole particle. With this linear dependence one can therefore only estimate the mass of the groundstate.

4. Excited States

Excited states, which in our model would represent higher families, are possible with more general non-linear dependencies $f(Q)$.

For the wave mechanical treatment of this problem it is convenient to use the 4-dimensional representation by making a canonical transformation

$$\boldsymbol{\vartheta}\cdot d\boldsymbol{v}+\vartheta_0\,d v_0+u_a\,dp_a=d\Phi(\boldsymbol{v},v_0,p_a) \tag{4.1}$$

with the generating function

$$\Phi=\frac{v_0}{\sqrt{1-v^2}}(\boldsymbol{v}\cdot\boldsymbol{\vartheta}+i\vartheta_4), \tag{4.2}$$

and where v_0, ϑ_0 are superfluous coordinates. Expressed in the new variables, one has

$$R=-\tfrac{1}{2}M_{ab}^2,\quad M_{ab}=u_a\vartheta_b-u_b\vartheta_a, \tag{4.3}$$

and with $P_a=\{\boldsymbol{P}, iH\}$ one finds for (3.16)

$$K=u_aP_a+\sqrt{-\dot{u}_a^2}\,F(R)=0. \tag{4.4}$$

Because $u_a^2=-1$ and $u_a p_a=0$, the superfluous coordinates v_0 and ϑ_0 can be eliminated. Putting

$$P_a=\frac{\hbar}{i}\frac{\partial}{\partial x_a},\quad p_a=\frac{\hbar}{i}\frac{\partial}{\partial u_a} \tag{4.5}$$

one obtains the wave equation

$$K\psi\equiv\left[\left(u_a,\frac{\hbar}{i}\frac{\partial}{\partial x_a}\right)+F(R)\right]\psi(x,u)=0, \tag{4.6}$$

where

$$R=-\frac{1}{2}M_{ab}^2,\quad M_{ab}=\frac{\hbar}{i}\left(u_a\frac{\partial}{\partial u_b}-u_b\frac{\partial}{\partial u_a}\right). \tag{4.7}$$

For $\boldsymbol{P}=0$, the wave function has the form

$$\psi(x,u)=\psi(u)\,e^{-i\varepsilon t/\hbar} \tag{4.8}$$

with the wave equation for $\psi(u)$

$$F(R)\,\psi(u)=\frac{\varepsilon}{\sqrt{1-v^2}}\psi(u), \tag{4.9}$$

or if G is the inverse function for F:

$$R\psi(u)=G\left(\frac{\varepsilon}{\sqrt{1-v^2}}\right)\psi(u). \tag{4.10}$$

From the condition $u_a p_a = 0$ follows that

$$R = -\hbar^2 \frac{\partial^2}{\partial u_a^2}. \tag{4.11}$$

With (θ, φ spherical polar coordinates)

$$u_a = [\sinh a \cdot \sin\theta \cdot \cos\varphi, \sinh a \cdot \sin\theta \cdot \sin\varphi, \sinh a \cdot \cos\theta, i\cosh a],$$

$$\psi = \psi_0/\sinh a, \quad \operatorname{tgh} a = v \tag{4.12}$$

the wave equation becomes

$$-\left[\frac{\partial^2}{\partial a^2} - 1 - \frac{M^2}{\sinh^2 a}\right]\psi_0 = G(\varepsilon\cosh a)\,\psi_0, \tag{4.13}$$

where

$$M^2 = (\boldsymbol{v}\times\boldsymbol{\vartheta})^2 = -\frac{1}{\sin\theta\,\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}, \tag{4.14}$$

having the eigenvalues $j(j+1)$, and where j is an integer. The wave equation, therefore, finally becomes

$$\frac{d^2\psi_0}{da^2} = V(a)\,\psi_0 = \left[1 + \frac{j(j+1)}{\sinh^2 a} - G(\varepsilon\cosh a)\right]\psi_0. \tag{4.15}$$

The eigenvalues can be obtained by the WKB method, with the factor $j(j+1)$ to be replaced by $(j+\frac{1}{2})^2$ to account for the singularity at $a=0$. The eigenvalues are then determined by the equation

$$J = \frac{1}{\pi}\int_{a_1}^{a_2}\sqrt{-V(a)}\,da = n + \tfrac{1}{2}, \quad n = 0, 1, 2, \ldots \tag{4.16}$$

with

$$V(a) = 1 + \frac{\left(j+\frac{1}{2}\right)^2}{\sinh^2 a} - G(\varepsilon\cosh a). \tag{4.17}$$

Of special interest are the cases where $j=-1/2$ because they correspond to the correct angular momentum quantization rule for the Zitterbewegung. For $j=-1/2$ one simply has

$$V(a) = 1 - G(\varepsilon\cosh a). \tag{4.18}$$

To obtain an eigenvalue requires a finite value of the phase integral (4.16). The function $G(x)$ ($x=\varepsilon\cosh a$), must therefore qualitatively have the form of a parabola cutting the line $G=1$ at two points x_1, x_2 in between which $G(x)>1$. One can then distinguish two limiting cases, the first one when $\varepsilon \ll 1$, and the second one when $\varepsilon \gg 1$. In both cases one may approximate

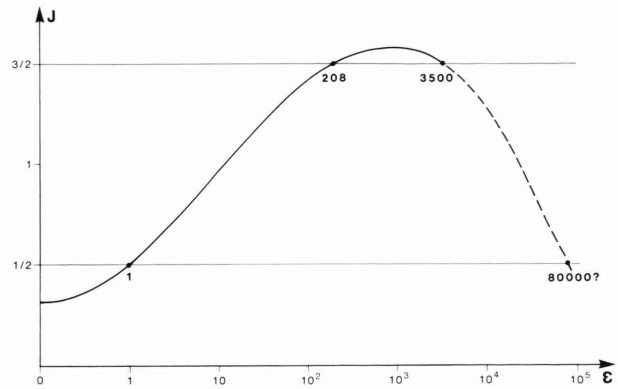

Fig. 2. Qualitative form of the phase integral $J=J(\varepsilon)$ to determine the number of families and their masses.

(4.16) as follows:

$$J \cong (1/\pi)\sqrt{-V(a)}\,(a_2 - a_1). \tag{4.19}$$

In the first case $a \gg 1$, and one has

$$a \cong \ln\left(\frac{x}{\varepsilon} + \sqrt{\frac{x^2}{\varepsilon^2} - 1}\right) \approx \ln\left(\frac{2x}{\varepsilon}\right) - \frac{\varepsilon^2}{4x^2} + \ldots, \tag{4.20}$$

hence

$$a_2 - a_1 \cong \ln\left(\frac{x_2}{x_1}\right) + \frac{x_2^2 - x_1^2}{x_1^2 x_2^2}\frac{\varepsilon^2}{4} + \ldots. \tag{4.21}$$

In the second case one has

$$a \cong \sqrt{2}\left(\frac{x}{\varepsilon} - 1\right)^{1/2}, \tag{4.22}$$

or if $x/\varepsilon \gg 1$ simply

$$a \cong \sqrt{2x/\varepsilon}, \tag{4.23}$$

hence

$$a_2 - a_1 \cong \sqrt{\frac{2}{\varepsilon}}\,(x_2^{1/2} - x_1^{1/2}). \tag{4.24}$$

One, therefore, sees that the phase integral has for $\varepsilon \ll 1$ the form $J = \boldsymbol{a} + \boldsymbol{b}\varepsilon^2$, but for $\varepsilon \gg 1$ the form $J = \boldsymbol{a}/\sqrt{\varepsilon}$. The $J(\varepsilon)$ curve can for this reason cut twice the lines $J=1/2$ ($n=0$), and $J=3/2$ ($n=1$).

In Fig. 2 we have adjusted the phase integral to account for the electron, muon and tau electron, where we have to set $\varepsilon = E/mc^2$, with m the electron mass. The fact that the mass ratio of the tau and muon are so much smaller than the mass ratio of the muon and electron, suggests that both the muon and tau result from cuts of the line $J=3/2$. Because of the proximity on the $J=3/2$-line, it is unlikely that the phase integral would cut the line $J=5/2$ or higher. Since for large values of ε, $J \propto 1/\sqrt{\varepsilon}$, it follows that there must be one more eigenvalue for which $J=1/2$, which from the position of the first three families is guessed to be around $80\,000\, mc^2 \cong 40$ GeV. Our result, therefore, strongly suggests that there are no more than four particle families. The recently reported maximum number of particle families deduced from the width of the Z^0 particle instead suggests that there are no more than three families [7]. This conclusion, however, depends on the assumption that the neutrinos of the different particle families have zero rest mass. In our model it is nevertheless still possible to obtain only three families by assuming that the phase integral $J=J(\varepsilon)$ cuts the line $J=1/2$ only once. For this to happen the phase integral would have to be shifted upwards to avoid the first cut at $J=1/2$. Another possibility is that the two points of intersection of the $J=3/2$-line coalesce into one point with the $J=3/2$-line tangent to the curve. Since we only use the general analytic form of the phase integral and try to adjust it to the masses of the known charged leptons, we can at best give upper and lower limits for the number of particle families, with the upper limit set equal to four. In the framework of the proposed substratum model a more definite conclusion has to await the determination of the structure function $f(Q)$ from the mass distribution of the exciton made up from the positive and negative mass vortex resonance. If the same quark-electron mass ratio ≈ 10 valid for the first family, also holds for the higher families, it would suggest that the quark masses belonging to the conjectured fourth family would be at ~ 400 GeV.

5. The Neutrino-Mass Problem

Since the simple pole-dipole model gives a surprisingly good value for the known mass ratio of the electron to the Planck mass, one must ask how the much smaller (by a factor of 10^6 or less) neutrino mass can be explained. In our model the electron is an exciton of two large quasiparticle masses of opposite sign, with its mass explained as the positive gravitational energy set up in between the interacting quasiparticles. It therefore seems logical to expect that the neutrino mass should likewise be explained as a gravitational field effect. We claim that a very small mass is possible by considering gravimagnetic fields. They, of course, occur in the framework of general relativity, but are here also possible because, very much like in superstring theories, elliptic deformations of the vortex rings lead to spin 2 tensorial waves [8]. In the simple pole-dipole configuration, the field responsible for producing the mass is a Newtonain-type gravistatic field. As already noticed by Hönl and Papapetrou [4], besides the pole-dipole configuration there exists a second fundamental configuration made up of two counterrotating vortices or rings of opposite mass. The gravimagnetic interaction leads there to a reduction of the energy from the still present gravistatic field energy with the reduction determined by the rotational velocity [9]. The rotational velocity, resp. value of $\gamma=(1=v^2/c^2)^{-1/2}$, by which the vortex resonances can move through the Planck aether as quasiparticles can be obtained by applying the uncertainty principle to their smallest spatial extension, which is about equal to the ring radius r_0. One has

$$\gamma m_\nu r_0 c \cong \hbar = m_p r_p c\,, \tag{5.1}$$

which in conjunction with $m_\nu c^2 \cong m_p c^2 (r_p/r_0)^2$ results in

$$\gamma \cong r_0/r_p\,. \tag{5.2}$$

For both the gravistatic and gravimagnetic interaction, the quasiparticles move with the same velocity given by (5.2), but where for two counterrotating quasiparticles the interaction energy is reduced by the factor γ^{-2}. Assuming that those quasiparticles, which interact both gravistatically and gravimagnetically, are otherwise behaving like a pole-dipole particle with the positive and negative mass components of the pole-dipole particle reduced by the factor $\gamma^{-2} = (r_p/r_0)^2$, one arrives at a value for the neutrino to electron mass ratio:

$$m_\nu/m \cong (r_p/r_0)^2 = (m_\varepsilon/m_p)^2\,. \tag{5.3}$$

With the above given number for $r_0/r_p = 6000$ one finds

$$m_\nu \cong 1.4 \times 10^{-2}\,\mathrm{eV}\,. \tag{5.4}$$

For the higher families the same ratio formula (5.3) might be true as well but because radial oscillations are there superimposed onto the simple pole-dipole configuration, the overall interaction is more complex, which could make (5.3) a poor approximation.

The hypothesis that the neutrinos are made up from a positive with a negative mass quasiparticle of the second fundamental rotating ring mode of the pole-dipole particle by Hönl and Papapetrou, would also explain why the neutrinos have no electric charge. It was shown [1] by the author that the Newtonian gravitational coupling of the Planck masses bound in the vortex filaments can be explained to result from their quantum mechanical zero point fluctuations. In a similar way, the electric charge can be understood to result from the zero point tilting fluctuations for the vortex rings [10]. It is plausible that in the rapidly rotating ring mode these tilting fluctuations cannot be excited. This finally would explain why the neutrino has a vanishing electric charge.

Appendix

For the linear dependence (3.18) we obtain from (3.7)

$$\frac{\mathrm{d}}{\mathrm{d}s}(2\gamma u_a + k_1 \ddot{u}_a) = 0 \tag{A.1}$$

or

$$2\dot{\lambda} u_a + 2\lambda \dot{u}_a + k_1 \dddot{u}_a = 0 \,. \tag{A.2}$$

Differentiating (3.3) with regard to s one has

$$u_a \dot{u}_a = 0, \quad u_a \ddot{u}_a + \dot{u}_a^2 = 0, \quad u_a \dddot{u}_a + 3\dot{u}_a \ddot{u}_a = 0, \tag{A.3}$$

by which (A.2) becomes

$$-2\dot{\lambda} - 3k_1 u_a \dddot{u}_a = -2\dot{\lambda} - \tfrac{3}{2} k_1 \frac{\mathrm{d}}{\mathrm{d}s}(\dot{u}_a^2) = 0 \,. \tag{A.4}$$

Summation over v gives

$$2\lambda = k_0 - (3/2)\, k_1 \dot{u}_v^2 \,, \tag{A.5}$$

where k_0 appears here as a constant of integration. Inserting (A.5) into (A.1), the Lagrange multiplier can be eliminated:

$$\frac{\mathrm{d}}{\mathrm{d}s}[(k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\, u_a + k_1 \ddot{u}_a] = 0 \,. \tag{A.6}$$

To show that (A.6) is the equation of motion for a pole-dipole particle we write it as follows:

$$\frac{\mathrm{d}P_a}{\mathrm{d}s} = 0, \quad P_a = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\, u_a + k_1 \ddot{u}_a \,, \tag{A.7}$$

where P_α are the components of the momentum-energy four-vector. For $k_1 = 0$ one has $P_\alpha = k_0 u_\alpha$, which by putting $k_0 = m$ is the four-momentum of a spinless particle with rest mass m.

The mass dipole moment is according to (3.8)

$$p_a = -k_1 \dot{u}_a \,, \tag{A.8}$$

as can be seen by the conservation of angular momentum

$$\frac{\mathrm{d}}{\mathrm{d}s} J_{ab} = 0 \,, \tag{A.9}$$

where

$$J_{ab} = [\boldsymbol{x}, \boldsymbol{P}]_{ab} + [\boldsymbol{p}, \boldsymbol{u}]_{ab} \tag{A.10}$$

and where $[\boldsymbol{x}, \boldsymbol{P}]_{ab} = x_a P_b - x_b P_a$. For a particle at rest $P_k = 0$, $k = 1, 2, 3$ one has

$$J_{kl} = [\boldsymbol{p}, \boldsymbol{u}]_{kl} = p_k u_l - p_l u_k, \quad k, l = 1, 2, 3 \,, \tag{A.11}$$

which is the spin angular momentum.

The energy of a pole-dipole particle at rest, and for which $u_4 = \gamma$, is determined by the fourth component

$$P_4 = im = i(k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\, \gamma \,, \tag{A.12}$$

but it can also be obtained from

$$\begin{aligned} P_a u_a &= -\gamma m = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\, u_a^2 + k_1 \ddot{u}_a u_a \\ &= -(k_0 - \tfrac{1}{2} k_1 \dot{u}_v^2) \,. \end{aligned} \tag{A.13}$$

The mass m therefore obeys the double equation

$$m = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\, \gamma = (k_0 - \tfrac{1}{2} k_1 \dot{u}_v^2)\, \gamma^{-1} \,. \tag{A.14}$$

To keep m finite in the limit $v \to 1$, resp. $\gamma \to \infty$, one must have

$$\begin{aligned} &\lim_{\gamma \to \infty} (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2) \to 0 \,, \\ &\lim_{\gamma \to \infty} (k_0 - \tfrac{1}{2} k_1 \dot{u}_v^2) \to \infty \,, \end{aligned} \tag{A.15}$$

which means that $k_0 \to (3/2)\, k_1 \dot{u}_v^2$ and $k_0 \to \infty$.

From (3.16) with $P_k = 0$, $k = 1, 2, 3$, follows for a circular orbit of radius r_c that

$$p = (k_0 - \tfrac{3}{2} k_1 \dot{u}_v^2)\, r_c \,, \tag{A.16}$$

or because of (A.12) that

$$p = m r_c / \gamma \,. \tag{A.17}$$

With $\boldsymbol{u} = \gamma \boldsymbol{v}$, one obtains for the spin angular momentum

$$J_z = -pu = -mvr_c \cong -mcr_c \,, \tag{A.18}$$

which is the same as (2.8).

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